

Aspects of Twistor Geometry and Supersymmetric Field Theories within Superstring Theory

Promotionsvortrag

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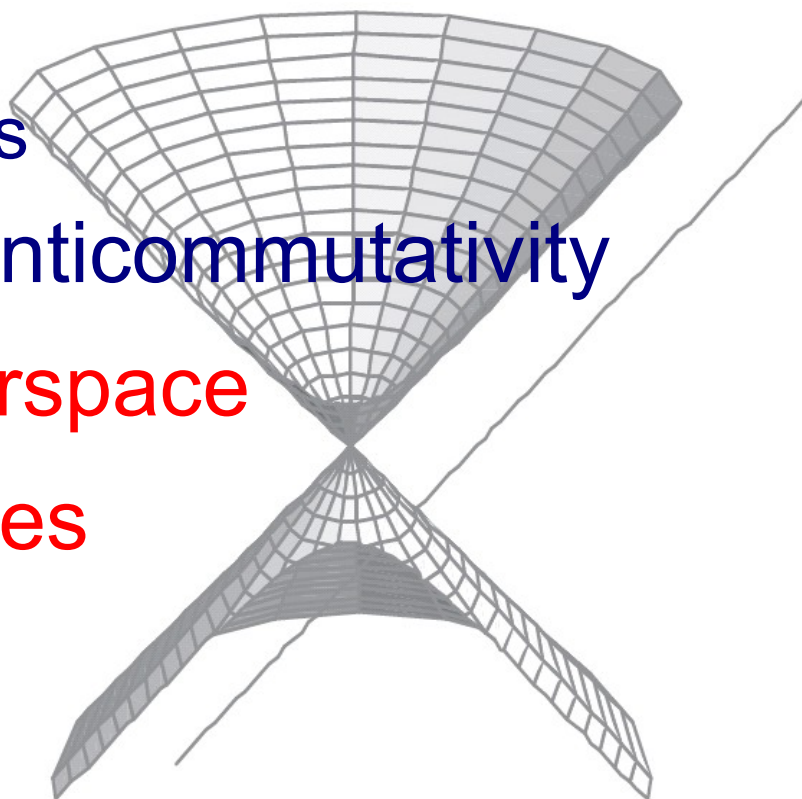


List of Publications

- with M. Wolf: *Constraint and super Yang-Mills equations on the deformed superspace $R_h^{4|16}$* , JHEP **0403** (2004) 048 [hep-th/0401147].
- with A.D.Popov: *On supertwistors, the Penrose-Ward transform and N=4 super Yang-Mills theory*, to appear in ATMP [hep-th/0405123].
- *The topological B-model on fattened complex manifolds and subsectors of N=4 self-dual Yang-Mills theory*, JHEP **0501** (2005) 042 [hep-th/0410292].
- with A.D. Popov and M. Wolf: *The topological B-model on a mini-supertwistor space and supersymmetric Bogomolny monopole equations*, JHEP **0510** (2005) 058 [hep-th/0505161].
- with M. Ihl: *Drinfeld twisted supersymmetry and non-anticommutative superspace*, JHEP **0601** (2006) 065 [hep-th/0506057].
- *On the mini-superambitwistor space and N=8 super Yang-Mills theory*, submitted to CMP, hep-th/0508137.
- with O. Lechtenfeld: *Matrix models and D-branes in twistor string theory*, submitted to JHEP, hep-th/0511130.

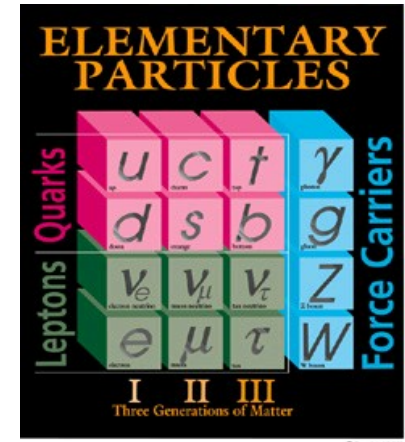
Outline

- ▶ Introduction
 - ▶ Extensions of Spacetime (**SUSY, NAC, Twistors**)
 - ▶ Gauge Theories
 - ▶ String Theory and D-Branes
- ▶ **Drinfeld Twists** and Non-Anticommutativity
- ▶ The **Mini-Superambitwistor space**
- ▶ **Matrix Models** and **D-Branes** in Twistor String Theory
- ▶ Conclusions



Extensions of Spacetime

- ▶ **Standard Model**: very successful!
but: evidently not down to arbitrary distances (e.g. **gravity**, **Landau pole**)
- ▶ Extensions of Spacetime:



- ▶ **Supersymmetry**: add fermionic dimensions

- ▶ **Noncommutativity**: $[\hat{x}^\mu, \hat{x}^\nu] \sim \Theta^{\mu\nu}$

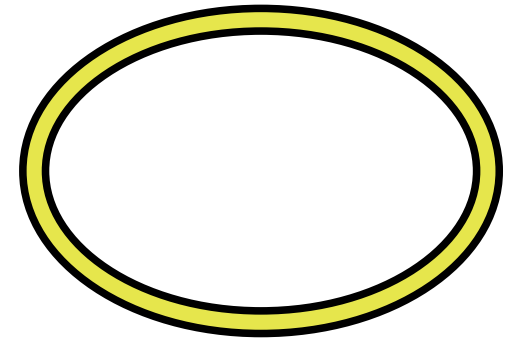
- ▶ **Both: Non-anticommutativity**: $\{\hat{\theta}^\alpha, \hat{\theta}^\beta\} \sim C^{\alpha\beta}$

- ▶ **Twistors**: add celestial spheres: $\mathbb{R}^{1,3} \rightarrow \mathbb{R}^{1,3} \times S^2$

Supersymmetry

$$Q|boson\rangle = |fermion\rangle$$

$$Q|fermion\rangle = |boson\rangle$$

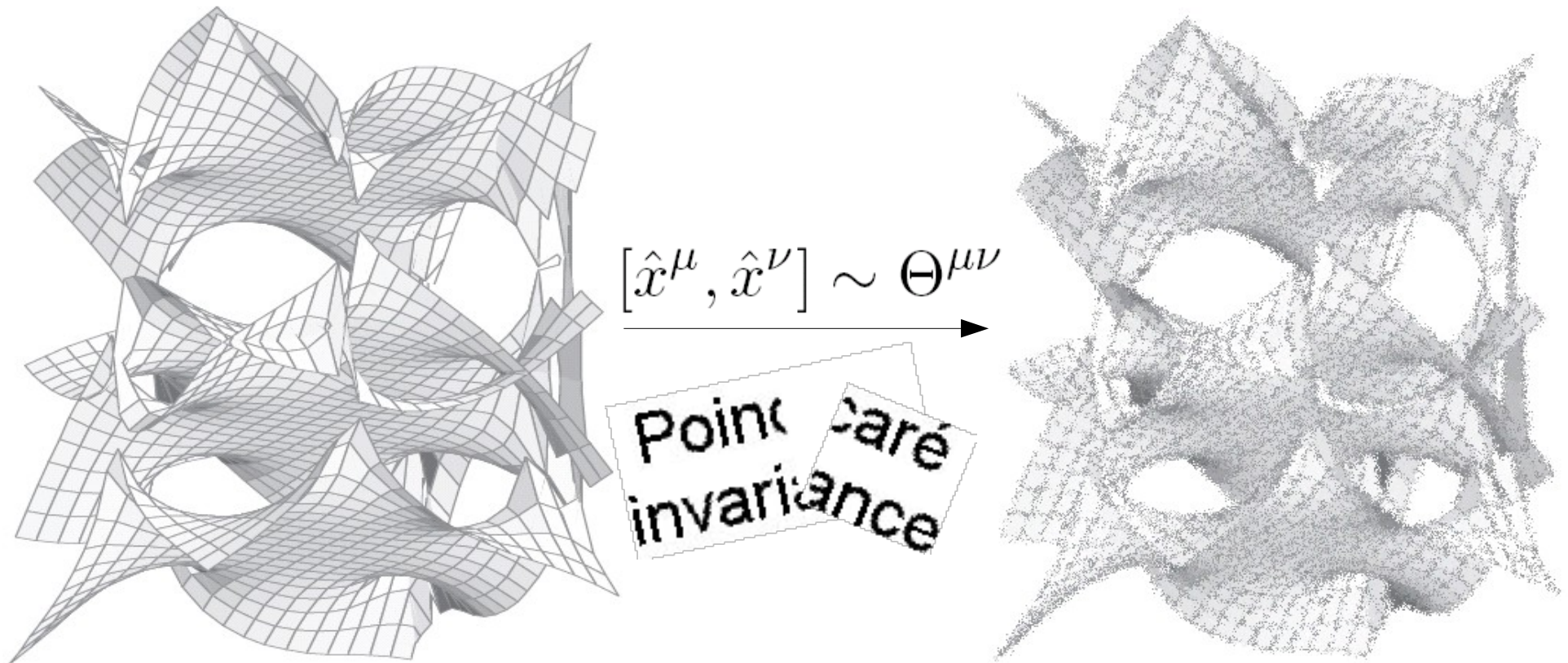


Q acts as translations in *fermionic* dimensions

- hierarchy problem solved
- unification of gauge couplings
- candidate for dark matter
- non-renormalization theorems
- upon localization, SUGRA appears
- possibly all particles in one multiplet
- leads to “nicer” string theories
- nice new mathematical structures
- SUSY is broken (e.g. “1/2” spectrum)
- Higgs should be found soon
- SUGRA non-renormalizable

Non-commutativity

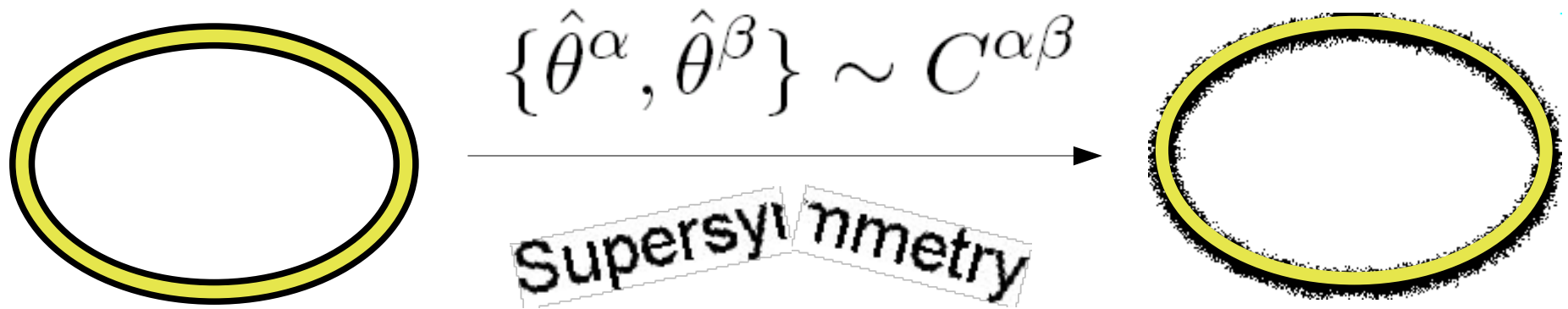
in various situations in **string theory** and **M-theory**



functions become operators on a Hilbert space
QFTs get **stringy features**, but **IR/UV** mixing

Non-anticommutativity

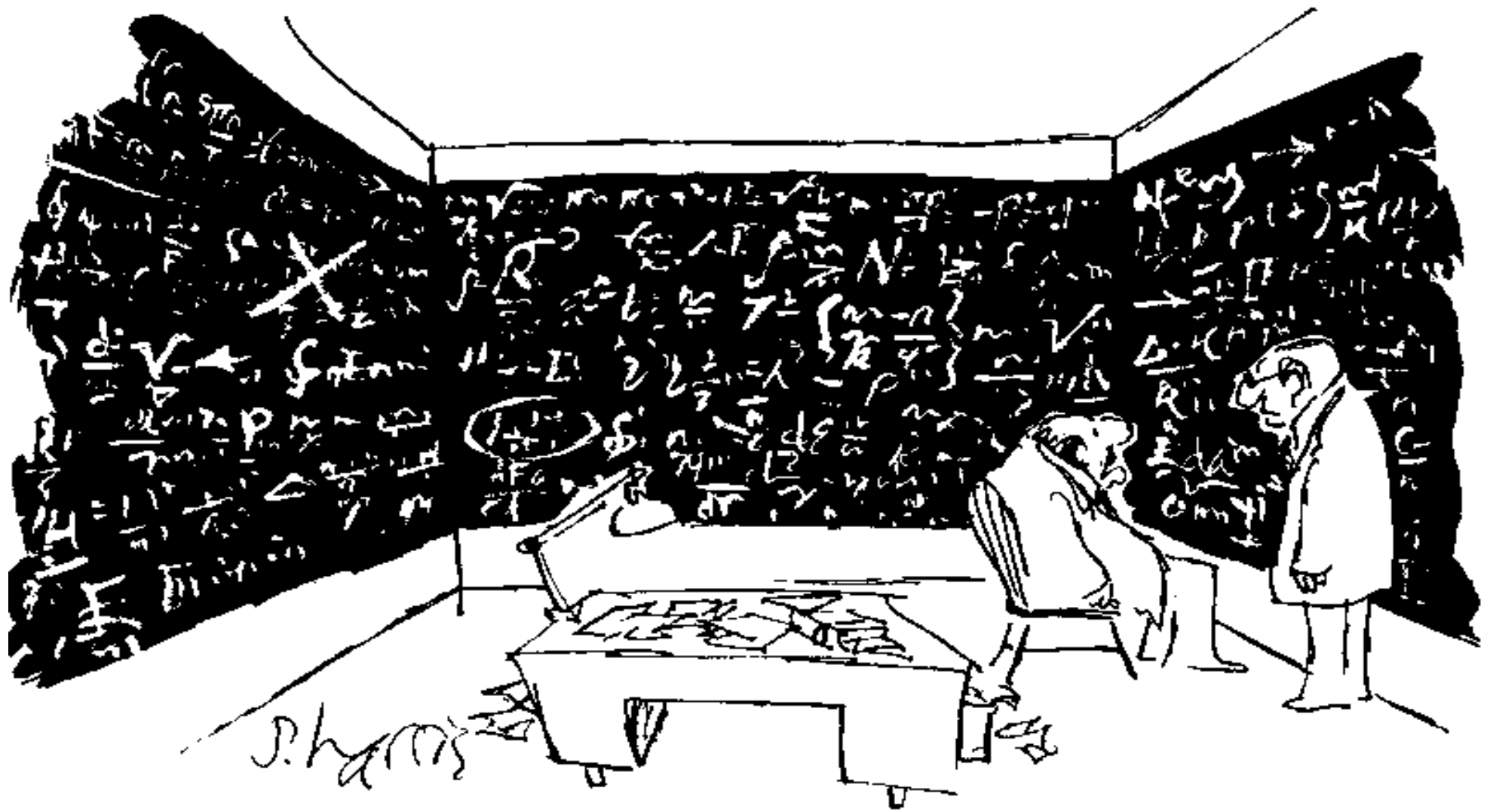
from graviphoton background in string theory



missing SUSY introduces problems:

- ▶ renormalizability hard to prove
- ▶ chiral rings and WT-Identities missing
- ▶ calculations much more involved

Summing up the situation...



"Whatever happened to *elegant* solutions?"

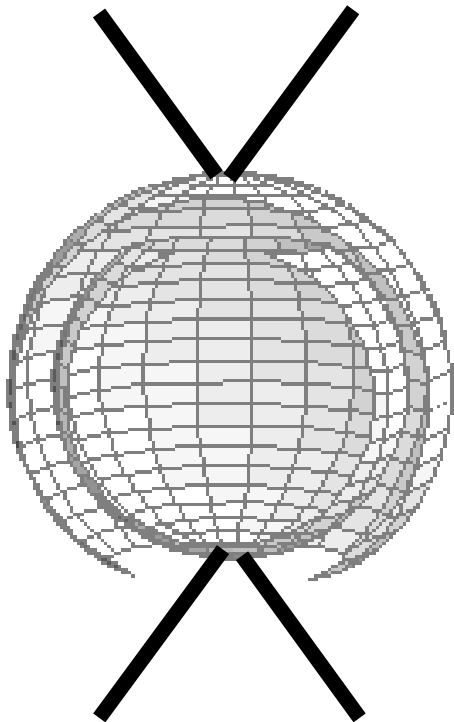
...but we will see that we can do better!

Twistors

add to every point in space its
 “celestial sphere” $\mathbb{R}^{1,3} \times S^2$

with a natural complex structure:

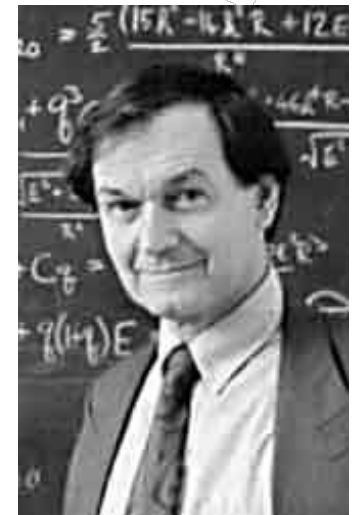
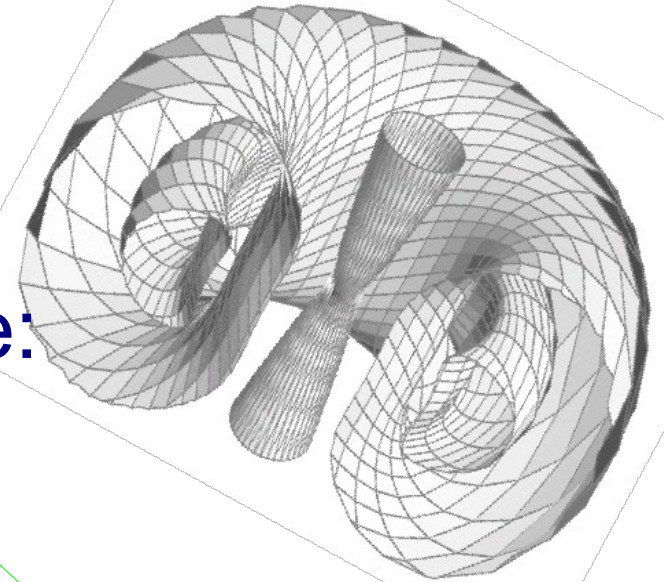
$$\mathcal{O}(1) \oplus \mathcal{O}(1) \rightarrow \mathbb{C}P^1$$



base: λ_+
 fibres: $z_+^\alpha = x^{\alpha 1} + \lambda_+ x^{\alpha 2}$

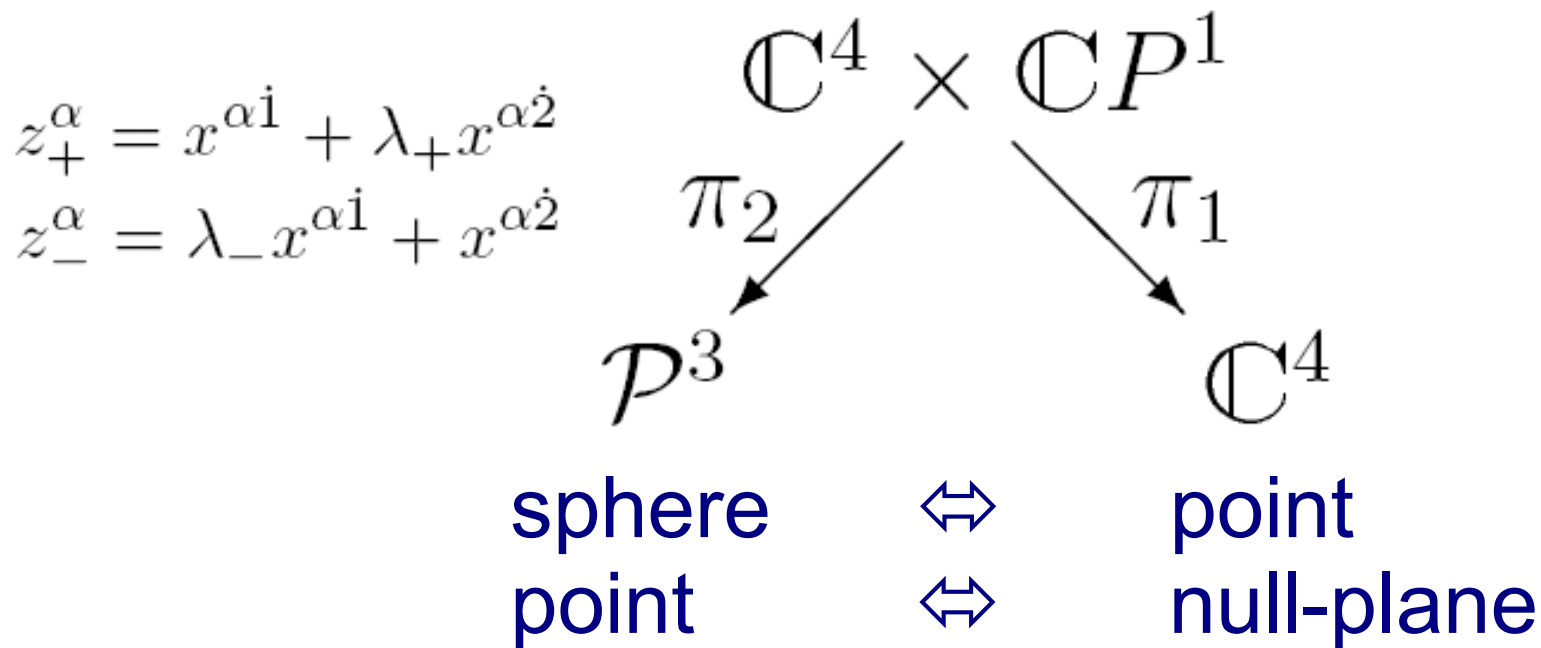
overlap: $\lambda_+ = (\lambda_-)^{-1}$
 $z_+^\alpha = \lambda_+ z_-^\alpha$

base: λ_-
 fibres: $z_-^\alpha = \lambda_- x^{\alpha 1} + x^{\alpha 2}$



Twistor correspondence

Complexify the picture:



Penrose-Ward transform:

holomorphicity
on twistor side



Integrability
on null-spaces



solutions to
Yang-Mills

Gauge Theories I

Maxwell equations: $F = dA$, $dF = 0$, $d*F = 0$

Field strength in spinor notation: $x^{\alpha\dot{\alpha}} = -i\sigma_{\mu}^{\alpha\dot{\alpha}}x^{\mu}$

$$[\nabla_{\alpha\dot{\alpha}}, \nabla_{\beta\dot{\beta}}] =: F_{\alpha\dot{\alpha}\beta\dot{\beta}} = \varepsilon_{\dot{\alpha}\dot{\beta}}f_{\alpha\beta} + \varepsilon_{\alpha\beta}f_{\dot{\alpha}\dot{\beta}}$$

Yang-Mills

$$\nabla_{\alpha\dot{\alpha}}F^{\alpha\dot{\alpha}\beta\dot{\beta}} = 0$$



Yang-Mills-Higgs

$$\nabla^{(\dot{\alpha}\dot{\beta})}F_{(\dot{\alpha}\dot{\beta})(\dot{\gamma}\dot{\delta})} = [\phi, \nabla_{(\dot{\gamma}\dot{\delta})}\phi]$$

$$\nabla^{(\dot{\alpha}\dot{\beta})}\nabla_{(\dot{\alpha}\dot{\beta})}\phi = 0$$

self-duality

$$f_{\dot{\alpha}\dot{\beta}} = 0$$



BPS

$$f_{\dot{\alpha}\dot{\beta}} = -\frac{i}{2}\nabla_{(\dot{\alpha}\dot{\beta})}\phi$$



Nahm and ADHM equations



Gauge Theories II

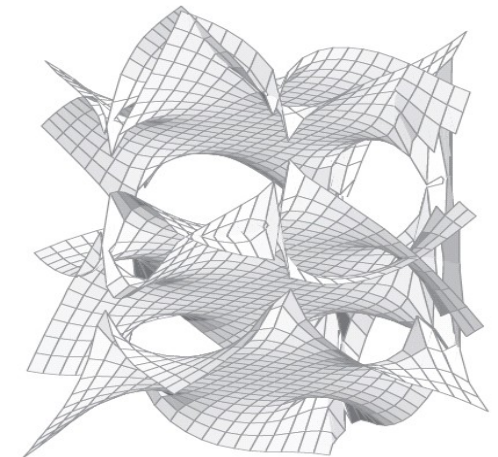
topological gauge theory: **Chern-Simons theory**

$$S_{\text{CS}} = \int_M \text{tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$
$$F = dA + A \wedge A = 0$$

Complex category: replace d with $\bar{\partial}$
holomorphic Chern-Simons theory:

$$S_{\text{hCS}} = \frac{1}{2} \int_M \Omega^{3,0} \wedge \text{tr} \left(A^{0,1} \wedge \bar{\partial} A^{0,1} + \frac{2}{3} A \wedge A \wedge A \right)$$
$$F^{0,2} = \bar{\partial} A^{0,1} + A^{0,1} \wedge A^{0,1} = 0$$

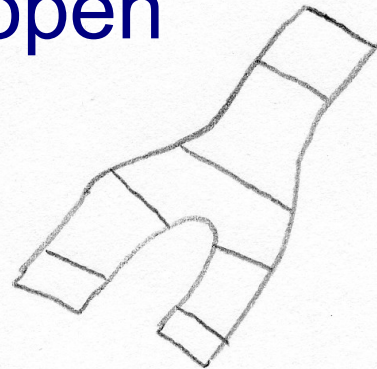
→ need Calabi-Yau manifolds



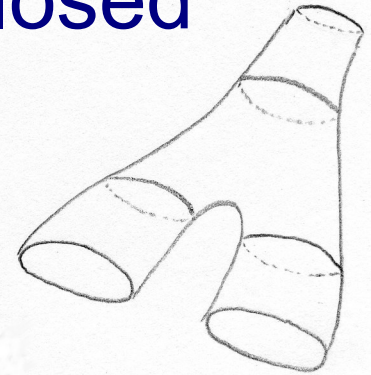
String Theory

String theory:
theory of embeddings
of 2d manifolds
into spacetime

open

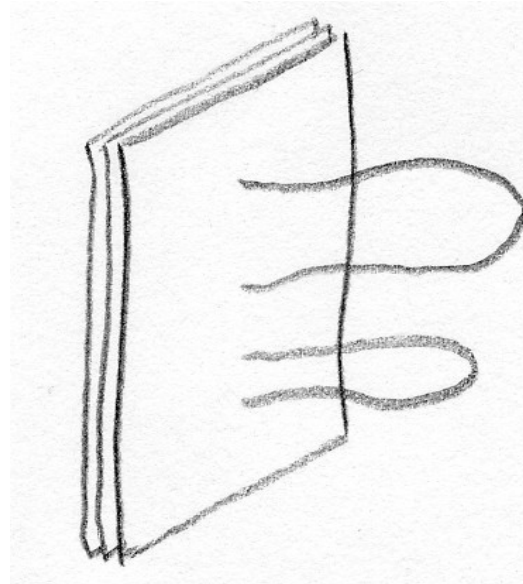
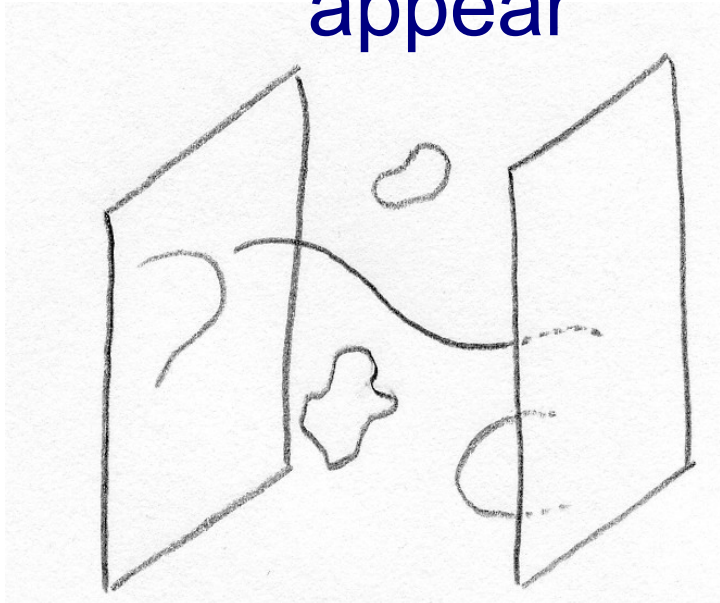


closed



inevitably, **D-branes**
appear

stack of n D-branes, ends
carry **gauge theory labels:**



$N=1$, $d=10$:

$U(n)$ SYM

$N=2$, $d=4$:

$U(n)$ SDYM

$N=2$, $d=6$ (top.):

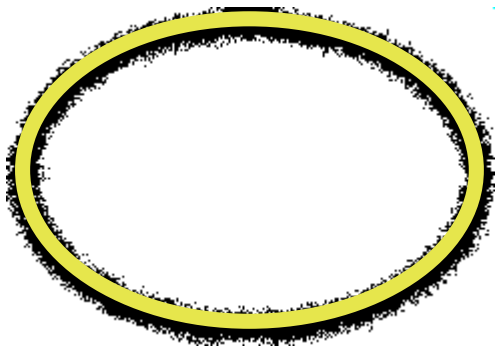
$GL(n, \mathbb{C})$ hCS

Drinfeld Twist

Drinfeld Twist

Model for:

$$\{\hat{\theta}^\alpha, \hat{\theta}^\beta\} \sim C^{\alpha\beta}$$



Consider the super Poincaré algebra as a Hopf algebra with coproduct $\Delta(x) = \mathbb{1} \otimes x + x \otimes \mathbb{1}$

Twist the coproduct:

$$\Delta^{\mathcal{F}}(Y) := \mathcal{F} \Delta(Y) \mathcal{F}^{-1}$$

$$\mathcal{F} = \exp \left(-\frac{\hbar}{2} C^{\alpha i, \beta j} Q_{\alpha i} \otimes Q_{\beta j} \right)$$

On the representation space (algebra of functions):

$$a \star b := m^{\mathcal{F}}(a \otimes b) := m \circ \mathcal{F}^{-1}(a \otimes b)$$

yielding $\{\theta^{\alpha i} \star \theta^{\beta j}\} = \hbar C^{\alpha i, \beta j}$ without destroying SUSY
(successfully introduced before)

Drinfeld Twist: Advantages



$$\{\theta^{\alpha i} \star, \theta^{\beta j}\} = \hbar C^{\alpha i, \beta j}$$

$$\Delta^{\mathcal{F}}(Y) := \mathcal{F} \Delta(Y) \mathcal{F}^{-1}$$

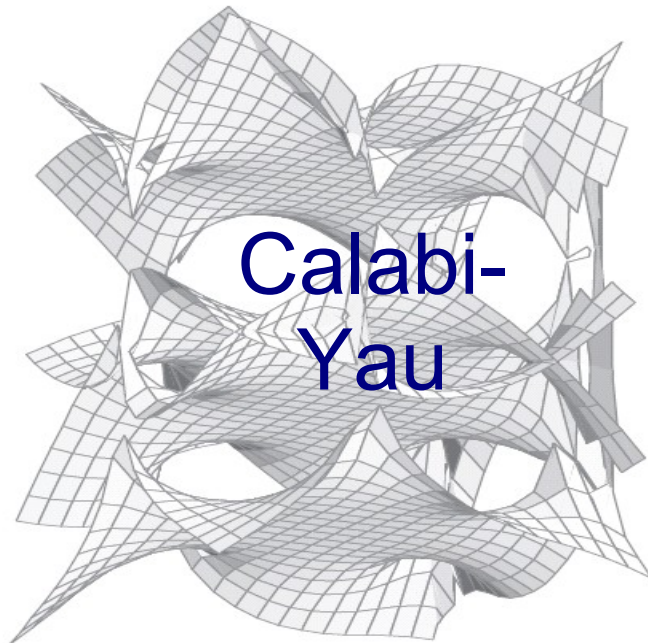
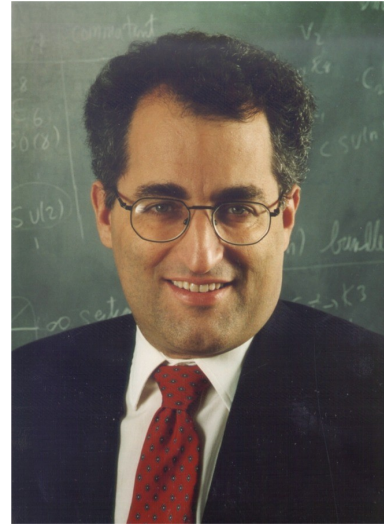
Twisted supersymmetry and chirality preserved!

- ✓ Representation content identical
- ✓ Vacuum energy 0 (in agreement with literature)
- ✓ SUSY chiral rings can be introduced
- ✓ Twisted Ward-Takahashi identities
- ✓ Non-renormalization?

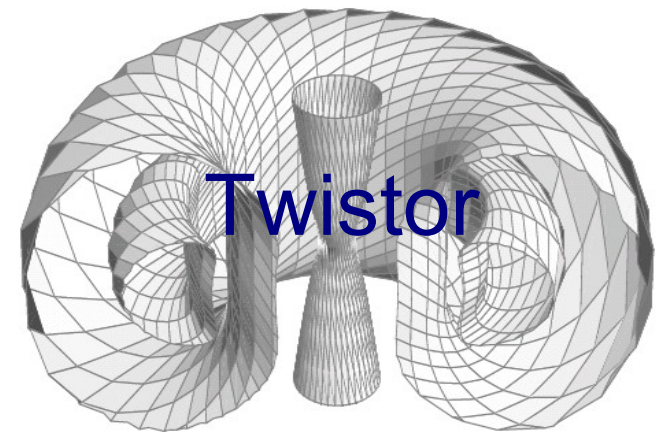
Naturalness argument looks promising.
(different suggestion from one-loop calculations)

Mini-Superambitwistors

Twistor String Theory



Calabi-
Yau



Twistor

Twistor String Theory

String theory on supertwistor space yields new string/gauge duality

top. B-model
on super CY



holomorphic
Chern-Simons



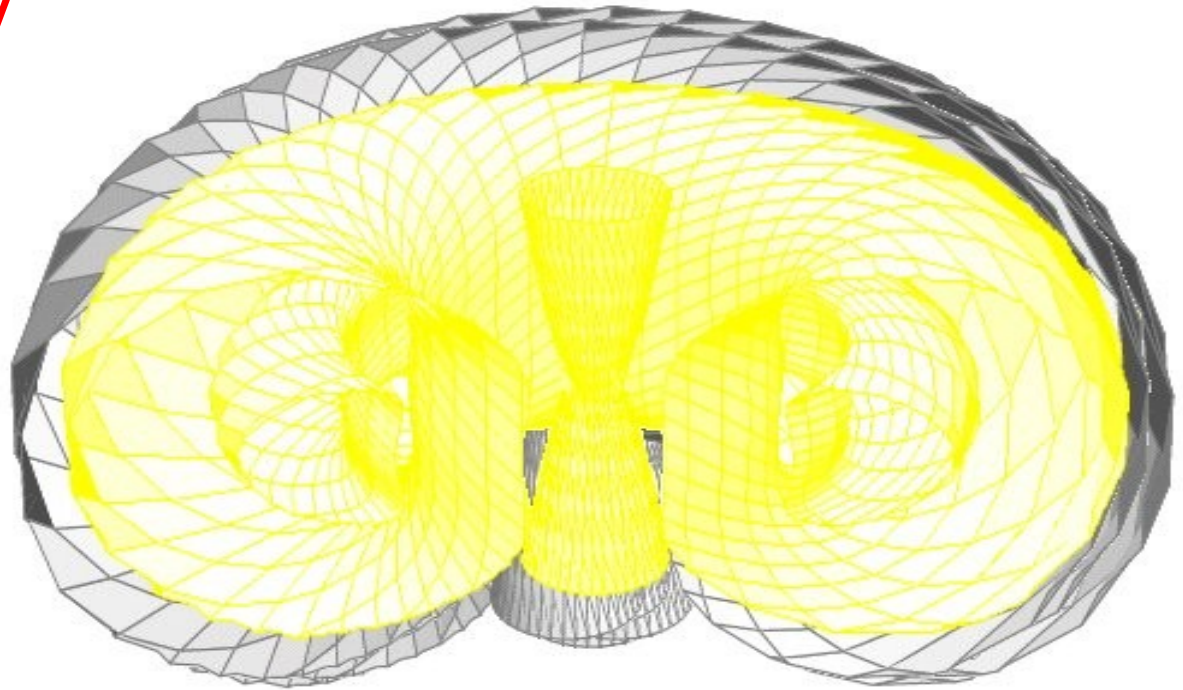
holomorphicity
on super CY



integrability
on null-spaces



solutions to
Yang-Mills



Supertwistor spaces

$$\mathcal{P}^{3|4} = \mathbb{C}^2 \otimes \mathcal{O}(1) \oplus \mathbb{C}^4 \otimes \Pi\mathcal{O}(1) \rightarrow \mathbb{C}P^1$$

quadric in $\mathcal{P}^{3|3} \times \mathcal{P}_*^{3|3}$

four dimensions:

N=4 SUSY SDYM

N=3,4 super YM

$$\mathcal{P}^{2|4} = \mathcal{O}(2) \oplus \mathbb{C}^4 \otimes \Pi\mathcal{O}(1) \rightarrow \mathbb{C}P^1$$

three dimensions:

N=8 Bogomolny

evidently missing:

quadric in $\mathcal{P}^{2|3} \times \mathcal{P}_*^{2|3}$

N=8 super YM

 Construction of mini-superambitwistor space

The Mini-Superambitwistor Space

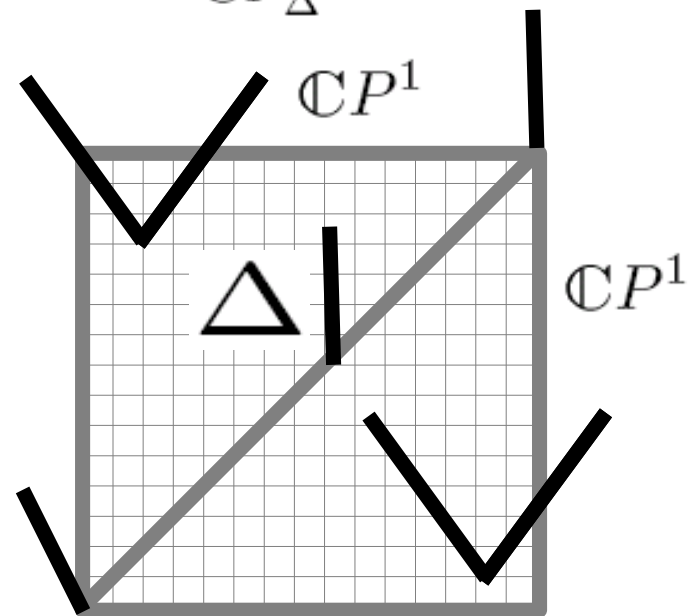
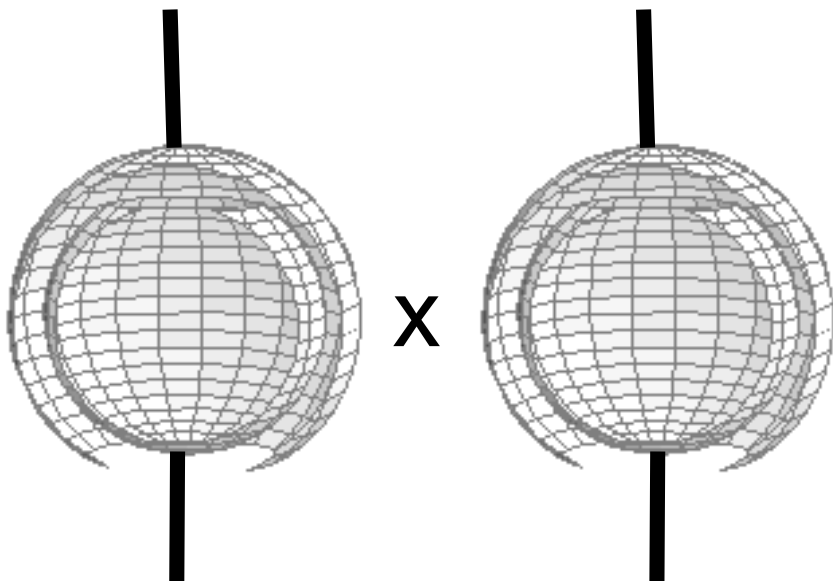
What is a quadric in $\mathcal{P}^{2|3} \times \mathcal{P}_*^{2|3}$?

$\mathcal{L}^{4|6}$

abstract definition:

$$0 \longrightarrow \mathcal{L}^{4|6} \longrightarrow \mathcal{P}^{2|3} \times \mathcal{P}_*^{2|3} \xrightarrow{\xi} \mathcal{O}_{\mathbb{C}P^1_\Delta}(2) \longrightarrow 0$$

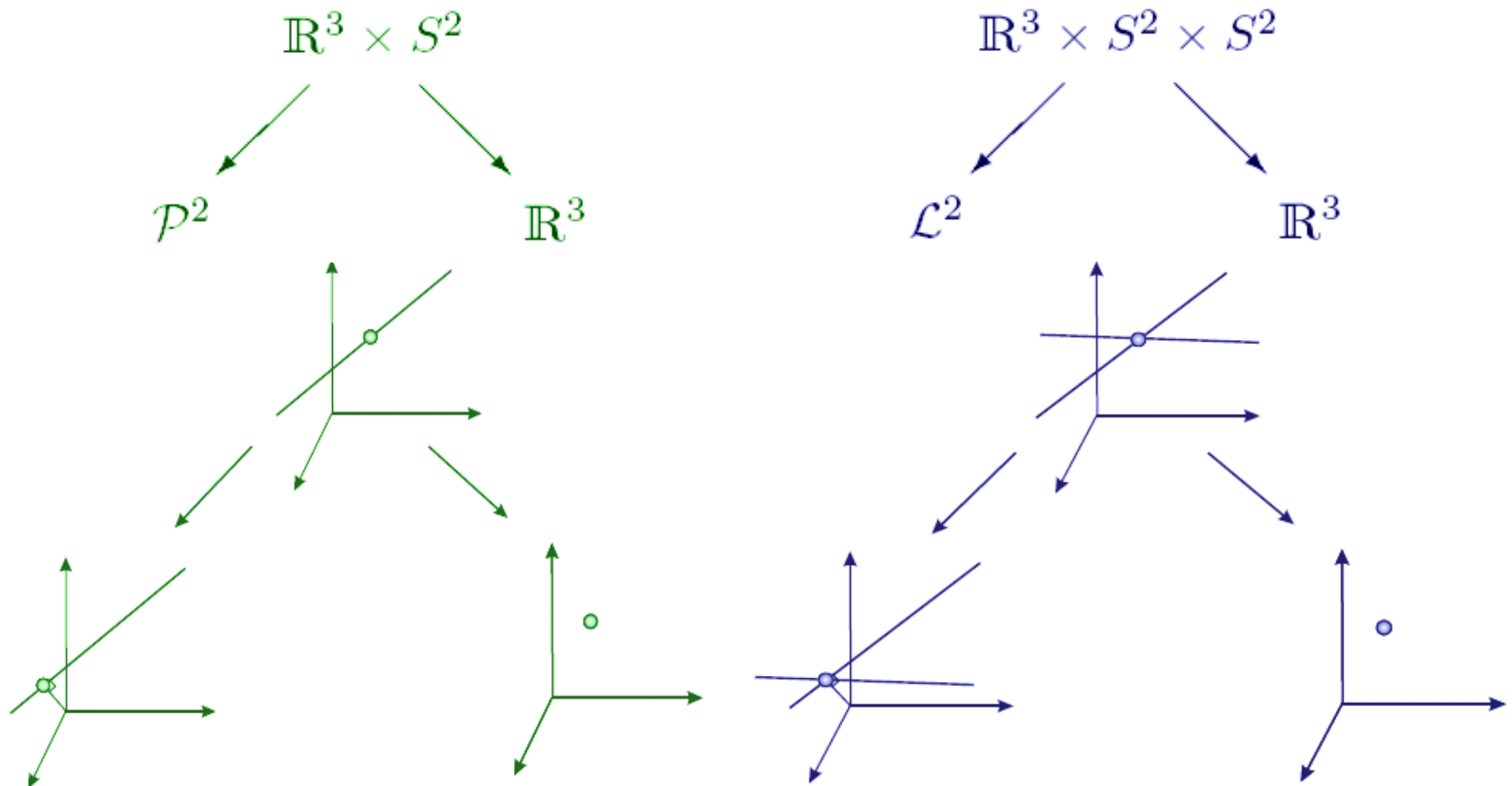
more explicitly:



fibration not vector bundle

Mini-Superambitwistor geometry

Interpretation via spaces of oriented lines:



Further remarks

after clarifying some technicalities: **PW works!**
(even for a bosonic truncation)

holomorphicity
on $L^{4|6}$



integrability
on null-lines



Yang-Mills-
Higgs

What about **topological B-model** and **twistor strings**?

Inconclusive:

- degeneracy cycles (\Rightarrow CY condition) OK
- unclear, how to define holomorphic Chern-Simons

Mirror conjecture:

$$\mathcal{P}^{2|4} \longleftrightarrow \mathcal{L}^{4|6}$$

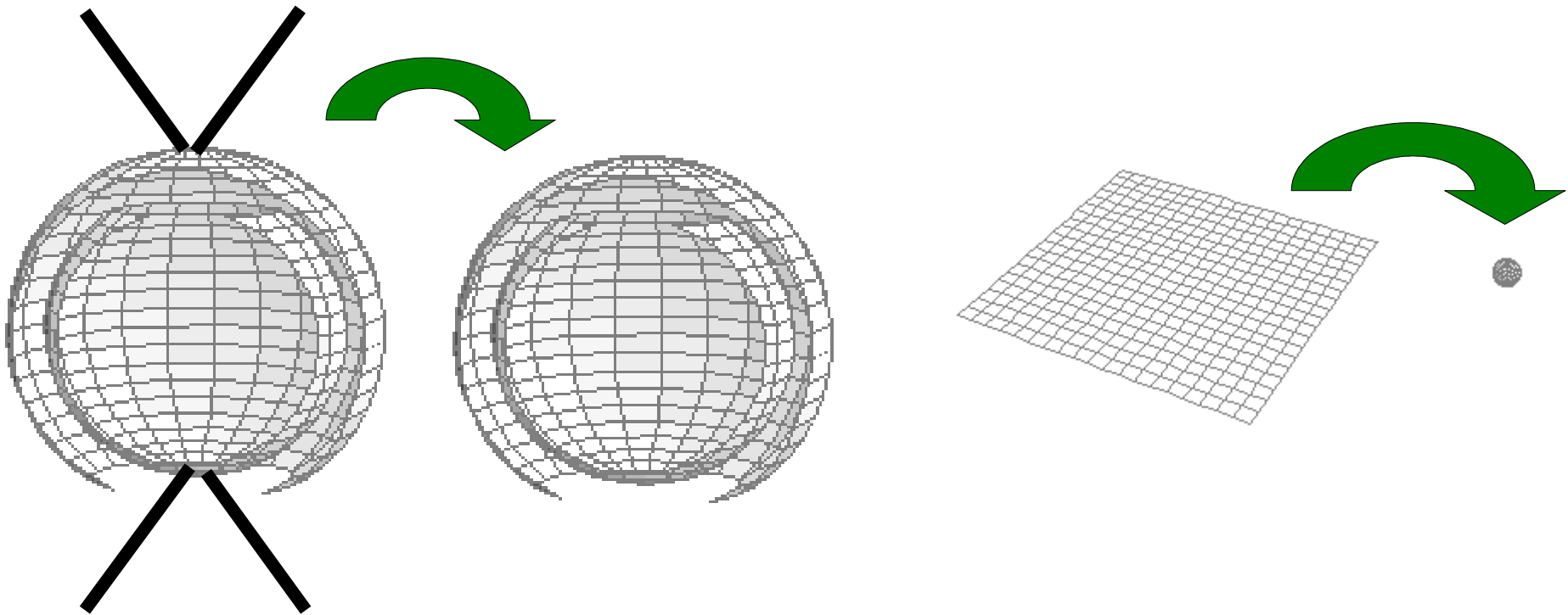
Twistor Matrix Models

Twistor Matrix Model

Matrix Models from dimensional reduction

Twistor side

Spacetime side



Penrose-Ward transform preserved:

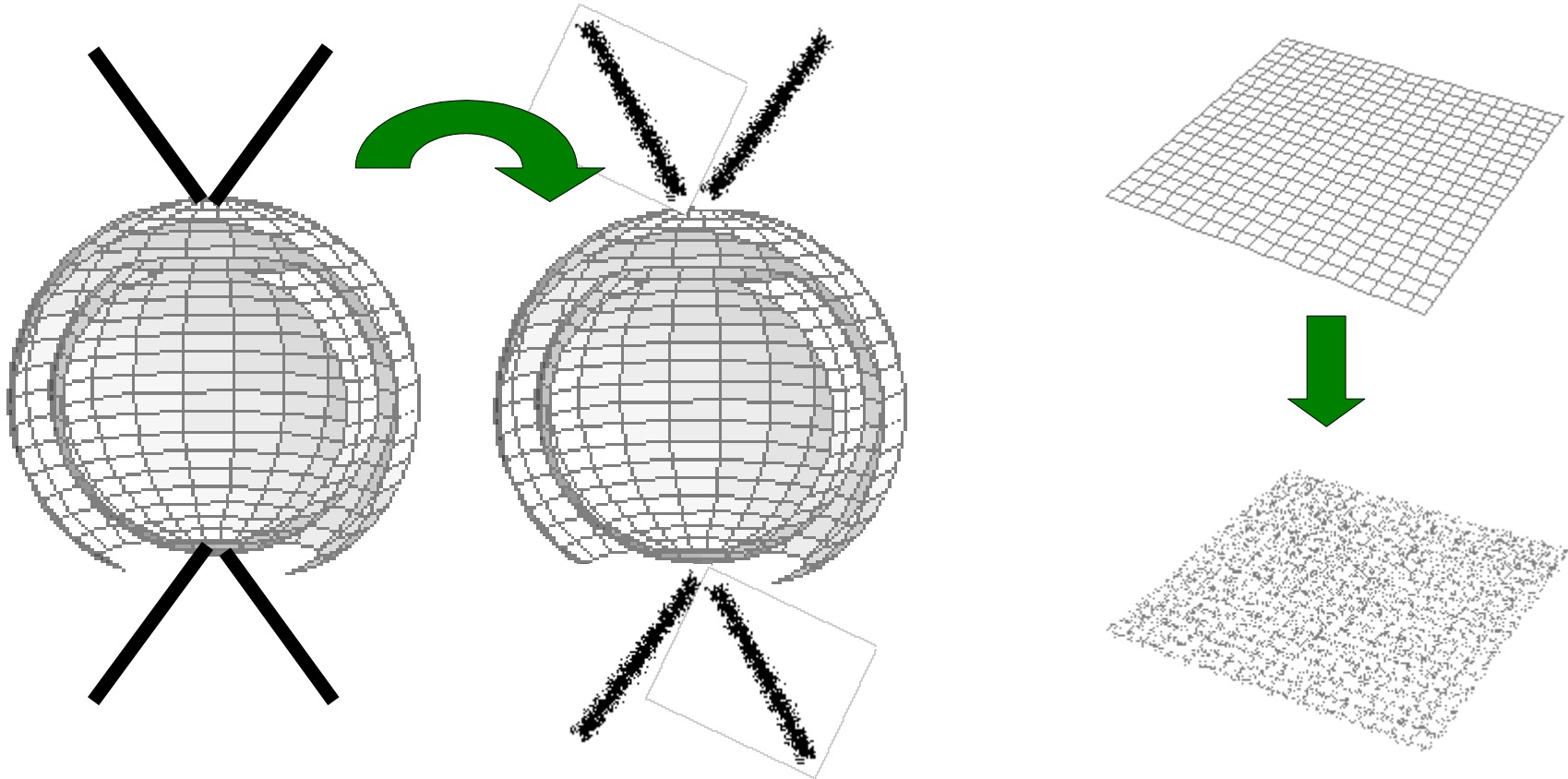
hCS MQM



SDYM MM

Alternative Matrix Models

Introduce noncommutativity:

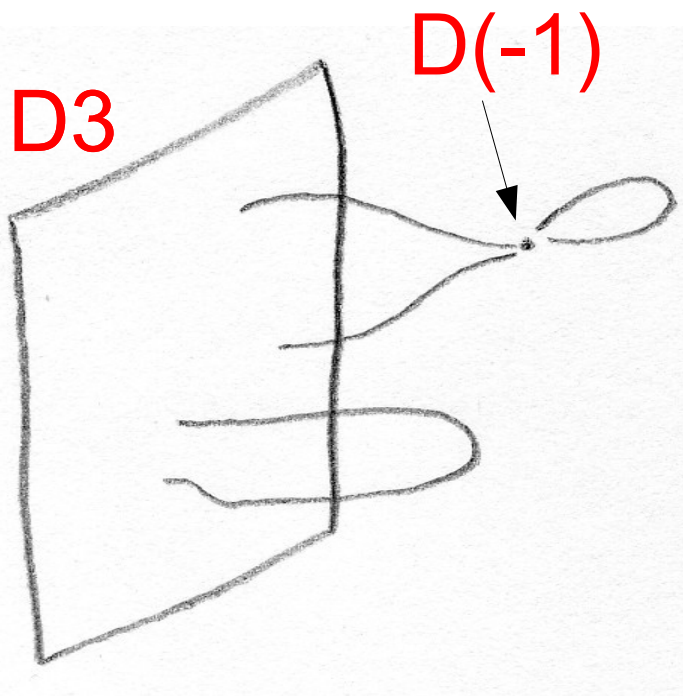


fields become operators/matrices,
derivatives become commutators,
integrals become traces,

First Results

- ▶ Matrix Models from dimensional reduction
 - ▶ SDYM MM describes $D(-1)$ -branes in $N=2$ ST
 - ▶ hCS MM describes $D1$ -branes in top. B-model
 - ▶ fermionic dimensions smeared out
- ▶ Matrix Models from noncommutativity
 - ▶ SDYM MM describes $D3$ -branes in $N=2$ ST in a B-field background
 - ▶ hCS MM describes $D5$ -branes in top. B-model in a B-field background
- ▶ use twistor methods to construct solutions

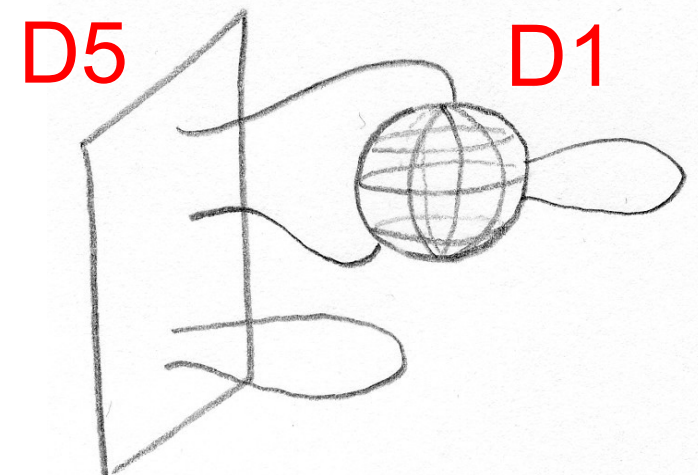
Further Results: ADHM eqns.



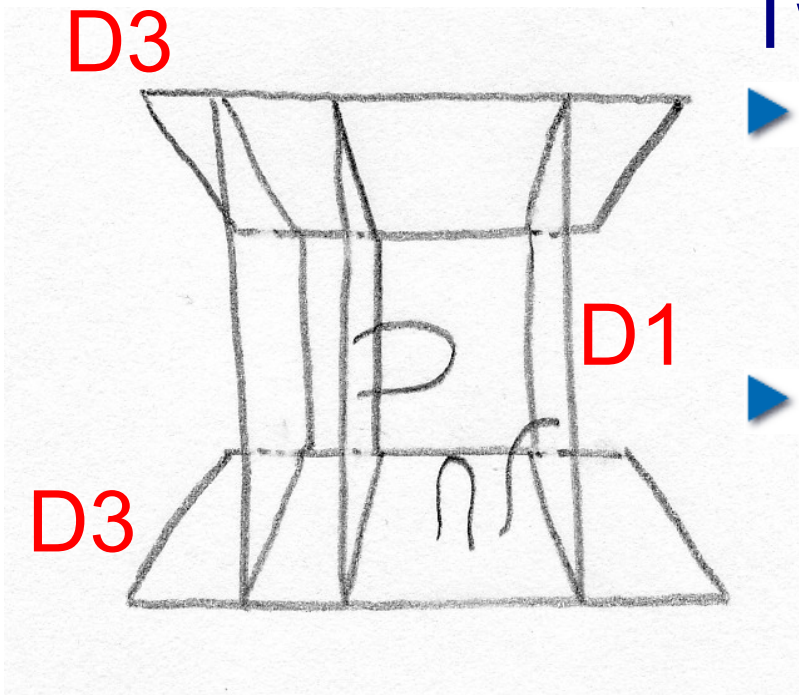
- Two equivalent descriptions:
- ▶ via **D3**-branes:
D(-1) is an instanton:
solution to **SDYM** ($ch_2 \neq 0$)
 - ▶ via **D(-1)**-brane:
all different strings build up to
ADHM eqns. (\approx **SDYM MM**)

in topological theory:

- ▶ via **D5**-branes:
D1 are solutions to **hCS**
- ▶ via **D1**-branes:
extension of **hCS MM**



Further Results: Nahm eqns.



Two equivalent descriptions:

- ▶ via **D3**-branes:

D1 is a monopole:
solution to **Bogomolny**

- ▶ via **D1**-brane:

D1-**D1**-strings build up to
Nahm eqns. (\approx **1d SYM**)

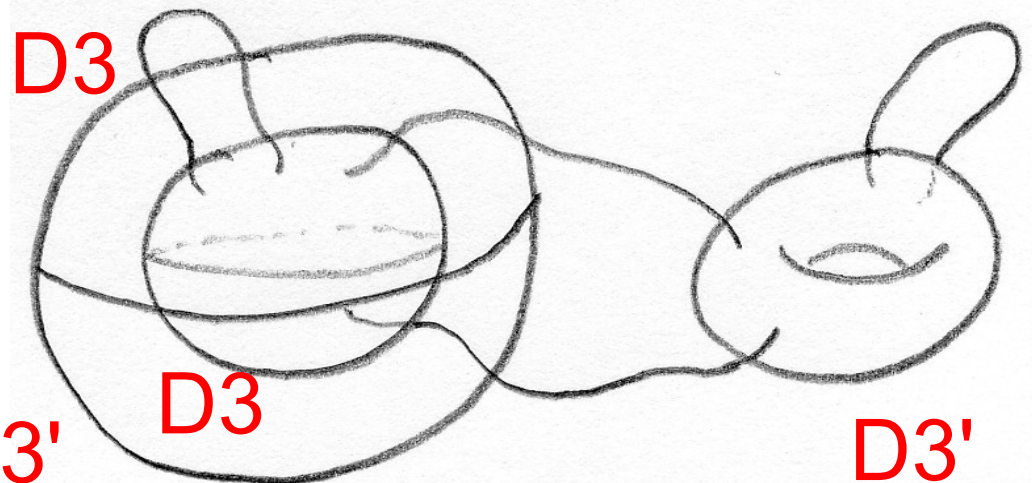
in topological theory:

- ▶ via **D3**-branes:

D3' solutions to **hCS**

- ▶ via **D3'**-branes:

reduction of **hCS** to **D3'**



Conclusions I

Drinfeld Twisted Non-Anticommutativity



$$\{\theta^{\alpha i} \star \theta^{\beta j}\} = \hbar C^{\alpha i, \beta j}$$

$$\Delta^{\mathcal{F}}(Y) := \mathcal{F} \Delta(Y) \mathcal{F}^{-1}$$

Done:

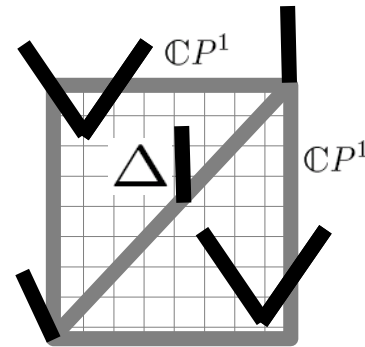
- ✓ SUSY NAC deformation
- ✓ consistency checks
- ✓ representation content
- ✓ reconstruction of chiral rings and WT-identities
- ✓ naturalness argument

Future directions:

- ✗ investigate situation non-renormalizability
- ✗ superconformal twist
- ✗ NAC supergravity
- ✗ convince people to use our approach

Conclusions II

The Mini-Superambitwistor Space

 $\mathcal{L}^{4|6}$ 

Done:

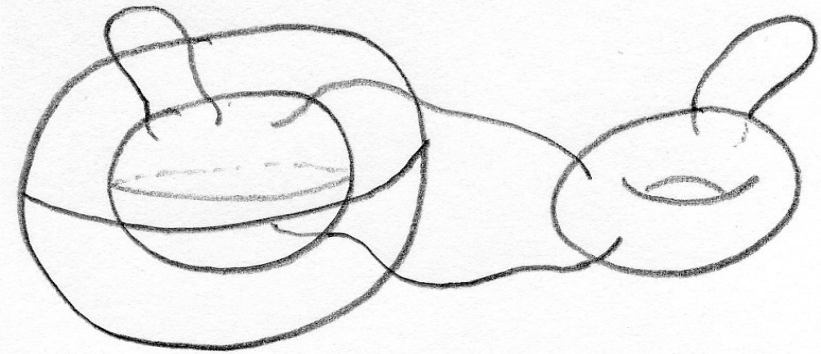
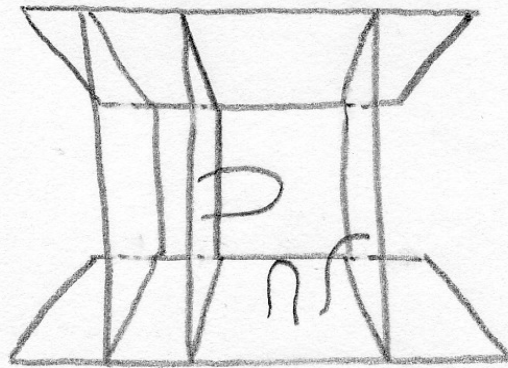
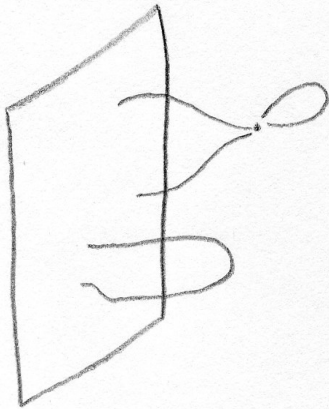
- ✓ Construction of mini-superambitwistor space
- ✓ clarification of geometry
- ✓ technicalities solved
- ✓ PW transform $N=8$
- ✓ PW transform $N=0$

Future directions:

- ✗ define hCS theory/
a topological B-model
- ✗ substantiate mirror
conjecture
- ✗ adapt recent construct.
of twistor actions

Conclusions III

Matrix Models & D-branes in Twistor String Theory



Done:

- ✓ definition of four MMs
- ✓ PW transform preserved
- ✓ D-brane interpretation
- ✓ extension to **ADHM**
- ✓ further MMs for **Nahm** equations

Future directions:

- ✗ clarify n to infinity
- ✗ study **Nahm** in detail
- ✗ mirror symmetry and T-duality
- ✗ carry over topological tools to **IIB** D-branes

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