$\mathcal{N}$=(1,0) SUSY in Six Dimensions, Self-dual Strings and Higher Instantons

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Workshop on Higher Gauge Theory and Higher Quantization

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Welcome to Edinburgh!

Live music - King Eider - Tonight 8:30

Outline

- Motivation for (1,0) models
- (1,0) models and '(1,0) gauge structures'
- Overlap with higher gauge theory
- Self-dual strings
    - abelian case
    - potential non-abelian analogues of 't Hooft-Polyakov monopole
- Higher instantons
    - potential non-abelian analogues of BPST instanton
- Conclusions

GOAL: (2,0) Theory
Looking for:

- SUSY transformations
- Equations of motion
- (Possibly a Lagrangian)
- Which algebraic structure describes multiple M5-branes?
  - which particular choice of vector spaces etc.?

(1,0) Models
[Samtleben,Sezgin,Wimmer]

- SUSY transformations ✓
- Equations of motion ✓ (different from twistor results)
- Lagrangian ✓ (PST-like)
- Which algebraic structure describes multiple M5-branes?
  - which particular choice of vector spaces etc.? ×
  - e.g. same as M2-branes? (probably not)

(1,0) Multiplets

(2,0) Multiplet

(B, X^I, Ψ)

(1,0) Tensor multiplet

$(B, X^6 = \phi, \Psi_L = \chi)$

(1,0) Hyper multiplet

(X^i, Ψ_R)

(1,0) Multiplets

(2,0) Multiplet
(B, X^I, Ψ)

(1,0) Tensor multiplet
$(B, X^6 = \phi, \Psi_L = \chi)$

(1,0) Hyper multiplet
$\cancel{(X^i, \Psi_R)}$

Auxiliary vector multiplet and 3-form field
(A, Y, λ) and C

Fields live in

$$\begin{array}{ccccc} \mathfrak{g}^* & \xrightarrow{\mathsf{g}} & \mathfrak{h} & \xrightarrow{\mathsf{h}} & \mathfrak{g} \\ C & & (B, \phi, \chi) & & (A, Y, \lambda) \end{array}$$

Maps:

$\mathsf{f} : \mathfrak{g} \wedge \mathfrak{g} \to \mathfrak{g}$

$\mathsf{d} : \mathfrak{g} \odot \mathfrak{g} \to \mathfrak{h}$

$\mathsf{b} : \mathfrak{h} \otimes \mathfrak{g} \to \mathfrak{g}^*$

Induced maps:

$\mathsf{f}^* : \mathfrak{g} \times \mathfrak{g}^* \to \mathfrak{g}^* \;, \quad \mathsf{d}^* : \mathfrak{h}^* \times \mathfrak{g} \to \mathfrak{g}^*$

For $\lambda \in \mathfrak{g}^*$, $\chi \in \mathfrak{h}$, $\gamma \in \mathfrak{g}$

$$\begin{aligned}
\mathsf{h}(\mathsf{g}(\lambda)) &= 0 \\
\mathsf{f}(\mathsf{h}(\chi),\gamma) - \mathsf{h}(\mathsf{d}(\mathsf{h}(\chi),\gamma)) &= 0 \\
\mathsf{f}(\gamma_{[1},\mathsf{f}(\gamma_2,\gamma_{3]})) - \tfrac{1}{3}\mathsf{h}(\mathsf{d}(\mathsf{f}(\gamma_{[1},\gamma_2),\gamma_{3]})) &= 0 \\
\mathsf{g}(\mathsf{b}(\chi_1,\mathsf{h}(\chi_2))) - 2\mathsf{d}(\mathsf{h}(\chi_1),\mathsf{h}(\chi_2)) &= 0 \\
\mathsf{g}(\mathsf{f}^*(\gamma,\lambda) - \mathsf{d}^*(\mathsf{h}^*(\lambda),\gamma) + \mathsf{b}(\mathsf{g}(\lambda),\gamma)) &= 0 \\
2(\mathsf{d}(\mathsf{h}(\mathsf{d}(\gamma_1,\gamma_{(2})),\gamma_{3)}) - \mathsf{d}(\mathsf{h}(\mathsf{d}(\gamma_2,\gamma_3)),\gamma_1)) & \\
-2\mathsf{d}(\mathsf{f}(\gamma_1,\gamma_{(2}),\gamma_{3)}) + \mathsf{g}(\mathsf{b}(\mathsf{d}(\gamma_2,\gamma_3),\gamma_1)) &= 0 \\
\mathsf{d}^*(\mathsf{h}^*(\mathsf{b}(\chi,\gamma_2)),\gamma_1) + \mathsf{b}(\chi,\mathsf{h}(\mathsf{d}(\gamma_1,\gamma_2))) & \\
+2\mathsf{b}(\mathsf{d}(\gamma_1,\mathsf{h}(\chi)),\gamma_2) - \mathsf{f}^*(\gamma_1,\mathsf{b}(\chi,\gamma_2)) & \\
-\mathsf{b}(\chi,\mathsf{f}(\gamma_1,\gamma_2)) - \mathsf{b}(\mathsf{g}(\mathsf{b}(\chi,\gamma_1)),\gamma_2) &= 0
\end{aligned}$$

This is a $\boxed{\text{(1,0)-gauge structure}}$

Example: Lie algebra $\mathsf{f}(\gamma_{[1},\mathsf{f}(\gamma_2,\gamma_{3]})) = 0$

SUSY transformations, E.O.M. and Gauge transformations

SUSY transformations

$$\delta A = -\bar{\varepsilon}\gamma\lambda \qquad \delta B = -\mathsf{d}(A, \bar{\varepsilon}\gamma\lambda) - \bar{\varepsilon}\gamma^{(2)}\chi$$

$$\delta\lambda^i = \tfrac{1}{4}\not{\mathcal{F}}\varepsilon^i - \tfrac{1}{2}Y^{ij}\varepsilon_j + \tfrac{1}{4}\mathsf{h}(\phi)\varepsilon^i \qquad \delta\chi^i = \tfrac{1}{8}\not{\mathcal{H}}\varepsilon^i + \tfrac{1}{4}\not{D}\phi\varepsilon^i - *\tfrac{1}{2}\mathsf{d}(\gamma\lambda^i, *\bar{\varepsilon}\gamma\lambda)$$

$$\delta Y^{ij} = -\bar{\varepsilon}^{(i}\not{D}\lambda^{j)} + 2\bar{\varepsilon}^{(i}\mathsf{h}(\chi^{j)}) \qquad \delta\phi = \bar{\varepsilon}\chi$$

$$\delta C = -\mathsf{b}(B, \bar{\varepsilon}\gamma\lambda) - \tfrac{1}{3}\mathsf{b}(\mathsf{d}(A, \bar{\varepsilon}\gamma\lambda)A) - \mathsf{b}(\phi, \bar{\varepsilon}\gamma^{(3)}\lambda)$$

where

$$\gamma = \gamma_\mu \mathrm{d}x^\mu,\ \gamma^{(2)} = \tfrac{1}{2}\gamma_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu,\ \gamma^{(3)} = \tfrac{1}{6}\gamma_{\mu\nu\rho}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \wedge \mathrm{d}x^\rho$$

$$\mathcal{F} = \partial A - \tfrac{1}{2}\mathsf{f}(A, A) + \mathsf{h}(B) \ \neq 0$$

$$\mathcal{H} = DB + \mathsf{d}(A, \partial A - \tfrac{1}{3}\mathsf{f}(A, A)) + \mathsf{g}(C) \neq 0$$

$$D = \partial - \mathsf{f}(A, \cdot) + \mathsf{h}(\mathsf{d}(A, \cdot))$$

or $\partial + 2\mathsf{d}(X, \mathsf{h}(\chi)) - \mathsf{g}(\mathsf{b}(\chi, X))$

or $\partial + \mathsf{f}^*(X, \lambda) - \mathsf{d}^*(\mathsf{h}^*(\lambda), X)$

Tensor multiplet E.O.M.

$$\begin{aligned}\mathcal{H} - \star\mathcal{H} &= -2\mathsf{d}(\bar{\lambda}, \gamma^{(3)}\lambda)\\ \not{D}\chi^i &= \mathsf{d}(\not{\mathcal{F}}, \lambda^i) + 2\mathsf{d}(Y^{ij}, \lambda_j) + \mathsf{d}(\mathsf{h}(\phi), \lambda^i) - 2\mathsf{g}(\mathsf{b}(\phi, \lambda^i))\\ D^2\phi &= 2\mathsf{d}(Y^{ij}, Y_{ij}) - *2\mathsf{d}(\mathcal{F}, *\mathcal{F}) - 4\mathsf{d}(\bar{\lambda}, \not{D}\lambda)\\ &\quad - 2\mathsf{g}(\mathsf{b}(\bar{\chi}, \lambda)) + 16\mathsf{d}(\bar{\lambda}, \mathsf{h}(\chi)) - 3\mathsf{d}(\mathsf{h}(\phi), \mathsf{h}(\phi))\end{aligned}$$

Anti-self-dual part constrained

Gauge transformations parametrized by (α, Λ, Ξ)

$$
\begin{aligned}
\delta A &= D\alpha - \mathsf{h}(\Lambda) \\
\delta B &= D\Lambda + \mathsf{d}(A, D\alpha - \mathsf{h}(\Lambda)) - 2\mathsf{d}(\alpha, \mathcal{F}) - \mathsf{g}(\Xi) \\
\delta C &= D\Xi + \mathsf{b}(B, D\alpha - \mathsf{h}(\Lambda)) - \tfrac{1}{3}\mathsf{b}(\mathsf{d}(D\alpha - \mathsf{h}(\Lambda), A), A) \\
&\qquad + \mathsf{b}(\Lambda, \mathcal{F}) + \mathsf{b}(\mathcal{H}, \alpha) + \dots
\end{aligned}
$$

Fake Curvature

$$\mathcal{F} = \partial A - \tfrac{1}{2}\mathsf{f}(A, A) + \mathsf{h}(B) \;\neq 0$$
$$\mathcal{H} = DB + \mathsf{d}(A, \partial A - \tfrac{1}{3}\mathsf{f}(A, A)) + \mathsf{g}(C) \neq 0$$

Fake curvature condition $\mathcal{F} = 0$, $\mathcal{H} = 0$, not SUSY invariant

unlike twistor construction, yet still large overlap for the algebraic structures

Overlap of (1,0)-gauge structures with familiar objects

Lie algebras $\subset$ (1,0)-gauge structures

$$0 \longrightarrow 0 \longrightarrow \mathfrak{g}$$

what about

$$0 \longrightarrow \mathfrak{h} \xrightarrow{\mathsf{h}} \mathfrak{g}$$

Answer: Courant-Dorfman algebras
(finite dimensional Courant algebroids)

Courant algebroid

Bundle E over manifold M with fiber metric $\langle \cdot, \cdot \rangle$, anchor map $\varrho : E \to TM$ and courant bracket $[\![\cdot, \cdot]\!]$ satistfying axioms

$$
\begin{aligned}
[\![[\![e_1, e_2]\!], e_3]\!] + [\![[\![e_2, e_3]\!], e_1]\!] + [\![[\![e_3, e_1]\!], e_2]\!] + \tfrac{1}{2}\mathcal{D}\big\langle [\![e_{[1}, e_2]\!], e_{3]} \big\rangle = 0 \\
\varrho([\![e_1, e_2]\!]) = [\varrho(e_1), \varrho(e_2)] \\
[\![e_1, f e_2]\!] = f[\![e_1, e_2]\!] + (\varrho(e_1)\cdot f)e_2 - \langle e_1, e_2\rangle \mathcal{D} f \langle \mathcal{D} f, \mathcal{D} g \rangle = 0 \\
\varrho(e)\cdot \langle e_1, e_2\rangle = \big\langle [\![e, e_1]\!] + \mathcal{D}\langle e, e_1\rangle, e_2 \big\rangle + \big\langle e_1, [\![e, e_2]\!] + \mathcal{D}\langle e, e_2\rangle \big\rangle
\end{aligned}
$$

where $\mathcal{D}$ is the pullback of the exterior derivative by the anchor map $\langle \mathcal{D} f, e\rangle := \frac{1}{2}\varrho(e)\cdot f$

$$
\mathcal{C}^\infty(M) \xrightarrow{\mathcal{D}} \Gamma(E)
$$

$$\mathfrak{h} \xrightarrow{\mathsf{t}} \mathfrak{g}$$

with $\mathsf{f} := -[\![\cdot,\cdot]\!]$, $\quad \mathsf{d} := \frac{1}{2}\langle\cdot,\cdot\rangle$

exactly a (1,0)-gauge structure of the form

$$0 \longrightarrow \mathfrak{h} \xrightarrow{\mathsf{t}} \mathfrak{g}$$

$$0 \longrightarrow \mathfrak{h} \xrightarrow{\mathsf{t}} \mathfrak{g}$$

$$\mu_1(\chi) := \mathsf{h}(\chi)\ , \qquad \mu_2(\gamma_1,\gamma_2) := -\mathsf{f}(\gamma_1,\gamma_2)\ ,$$
$$\mu_2(\gamma,\chi) := \mathsf{d}(\gamma,\mathsf{h}(\chi))\ , \quad \mu_3(\gamma_1,\gamma_2,\gamma_3) : \mathsf{d}(\gamma_{[1},\mathsf{f}(\gamma_2,\gamma_{3]}))\ ,$$

Courant-Dorfman algebras $\subset$ Lie 2-algebras

$$\boxed{NON-EXAMPLE : OCTONIONS}$$

$\mathsf{d}(\cdot,\cdot)$ cannot be written in terms of Lie 2-algebra products

$$0 \longrightarrow \mathbb{R} \longrightarrow \mathfrak{g}$$

$\mathsf{d} := \frac{1}{2}\langle\cdot,\cdot\rangle$ metric/Killing form

$\mathsf{f} := -[\cdot,\cdot]$

$\mathsf{g} = \mathsf{h} = \mathsf{b} = 0$

E.O.M.

$$\begin{aligned}
\mathcal{H}^- &= -\left\langle\bar{\lambda},\gamma^{(3)}\lambda\right\rangle \ , \\
\not{\partial}\chi^i &= \langle\not{\mathcal{F}},\lambda^i\rangle + 2\left\langle Y^{ij},\lambda_j\right\rangle \ , \\
\partial^2\phi &= 2\left\langle Y^{ij},Y_{ij}\right\rangle - *2\left\langle\mathcal{F},*\mathcal{F}\right\rangle - 4\left\langle\bar{\lambda},\not{\partial}\lambda\right\rangle \ ,
\end{aligned}$$

M2-brane 3-algebras
as Lie 2-algebras (Patricia will talk more about this)

$$\mathcal{A} \quad \longrightarrow \quad \mathfrak{g}_{\mathcal{A}}$$

$$\mu_2(g, a) = g \rhd a \ , \quad \mu_1 = \mu_3 = 0$$

Examples

BLG uses A_4 defined by $[e^\mu, e^\nu, e^\rho] = \varepsilon^{\mu\nu\rho\sigma} e^\sigma$

$$\mathbb{R}^4 \quad \longrightarrow \quad \mathfrak{so}(4)$$

ABJM uses

$$\mathsf{Mat}_{\mathbb{C}}(N) \quad \longrightarrow \quad \mathfrak{u}(N) \times \mathfrak{u}(N)$$

Problem:

if $\mu_2(\gamma,\chi) = \mathsf{d}(\gamma,\mathsf{h}(\chi))$

$$\mu_1 = \mathsf{h} = 0$$

then $\mu_2(\gamma,\chi) = 0$

$$0 \longrightarrow \mathcal{A} \longrightarrow \mathcal{A} \times \mathfrak{g}_{\mathcal{A}} \ ,$$

and choose the maps

$$\mathsf{g} = \mathsf{b} = 0 \ , \quad \mathsf{h}(v) = \begin{pmatrix} v \\ 0 \end{pmatrix} \ ,$$

$$\mathsf{d}\left(\begin{pmatrix} v_1 \\ g_1 \end{pmatrix}, \begin{pmatrix} v_2 \\ g_2 \end{pmatrix}\right) = \tfrac{1}{2}(g_1 \rhd v_2 + g_2 \rhd v_1) \ ,$$

$$\mathsf{f}\left(\begin{pmatrix} v_1 \\ g_1 \end{pmatrix}, \begin{pmatrix} v_2 \\ g_2 \end{pmatrix}\right) = \begin{pmatrix} \tfrac{1}{2}(g_2 \rhd v_1 - g_1 \rhd v_2) \\ [g_1, g_2] \end{pmatrix} \ ,$$

for $v \in \mathcal{A}$, $\left(\begin{smallmatrix} v_i \\ g_i \end{smallmatrix}\right) \in \mathcal{A} \times \mathfrak{g}_{\mathcal{A}}$.

Tensor gauge field with GxG symmetry [Chu]
as (1,0)-gauge structure

ABJM with boundary suggests $\mathfrak{u}(N) \times \mathfrak{u}(N)$ M5-brane symmetry

$$0 \longrightarrow \mathfrak{g} \longrightarrow \mathfrak{g} \times \mathfrak{g} ,$$

and choose the maps

$$\mathsf{h}(g) = \begin{pmatrix} -g \\ g \end{pmatrix} , \quad \mathsf{d}\left(\begin{pmatrix} g_1 \\ g_2 \end{pmatrix}, \begin{pmatrix} g_3 \\ g_4 \end{pmatrix} \right) = \tfrac{1}{2}([g_1, g_4] + [g_3, g_2]) ,$$

$$\mathsf{f}\left(\begin{pmatrix} g_1 \\ g_2 \end{pmatrix}, \begin{pmatrix} g_3 \\ g_4 \end{pmatrix} \right) = \begin{pmatrix} -[g_1, g_3] - \frac{1}{2}([g_1, g_4] - [g_3, g_2]) \\ -[g_2, g_4] - \frac{1}{2}([g_1, g_4] - [g_3, g_2]) \end{pmatrix} ,$$

If ker(g)=0

$$\mathfrak{g}^* \xrightarrow{\mathsf{g}} \mathfrak{h} \xrightarrow{\mathsf{h}} \mathfrak{g} \subset \text{Lie 3-algebras}$$

one parameter embedding

Using Q-manifolds

$$\mathfrak{g}^* \backslash \ker(\mathsf{g}) \xrightarrow{\mathsf{g}} \mathfrak{h} \xrightarrow{\mathsf{h}} \mathfrak{g} \subset \text{Lie 3-algebras}$$

corresponds to a particular parameter value

If fake curvature condition imposed

gauge transformations can be written in terms of Lie 3-algebra products

$$
\begin{aligned}
\delta A &= \partial\alpha + \mu_2(A,\alpha) - \mu_1(\Lambda)\ , \\
\delta B &= \partial\Lambda + \mu_2(B,\alpha) + \mu_2(A,\Lambda) + \tfrac{1}{2}\mu_3(A,A,\alpha) - \mu_1(\Xi)\ , \\
\delta C &= \partial\Xi + \mu_2(C,\alpha) + \mu_2(B,\Lambda) + \mu_2(A,\Xi) - \tfrac{1}{2}\mu_3(A,A,\Lambda) \\
&\qquad + \mu_3(B,A,\alpha) + \tfrac{2}{3}\mu_4(A,A,A,\alpha).
\end{aligned}
$$

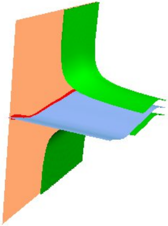
$$
\begin{aligned}
\mathcal{F} &= \partial A + \tfrac{1}{2}\mu_2(A,A) + \mu_1(B) = 0\ , \\
\mathcal{H} &= \partial B + \mu_2(A,B) + \tfrac{1}{6}\mu_3(A,A,A) + \mu_1(C) = 0\ ,
\end{aligned}
$$

But SUSY and E.O.M. cannot be written in terms of Lie 3-algebra products

$$
\begin{aligned}
\mathcal{H} &= \star D\phi \ , \\
\mathcal{F} &= \star \mathcal{F} \ , \\
\mathsf{h}(\phi) &= 0
\end{aligned}
$$

Differ from twistor equations

$$\mathcal{F} \neq 0$$



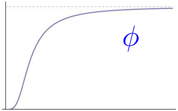
$$\phi = \frac{q}{|x|^2}$$

$0 \to \mathbb{R} \to 0$

(1,0)-models and twistor match

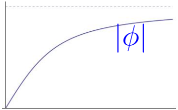
Suggests that M2-brane models should describe fuzzy S^3
- open problem

Akyol-Papadopolous Self-dual string



$$0 \to \mathbb{R} \to \mathfrak{su}(2)$$
$$\mathfrak{string}(\mathfrak{su}(2))$$

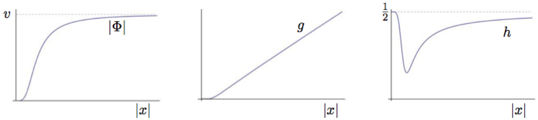
Compare to 't Hooft-Polyakov monopole



$$\phi = x^i e^i f(|x|)$$

$$\Phi = e_\mu x^\mu \; f(|x|) \; ,$$
$$B_{\mu\nu} = \varepsilon_{\mu\nu\kappa\lambda} e^\kappa x^\lambda \; g(|x|) \; ,$$
$$A_\mu = \varepsilon_{\mu\nu\kappa\lambda} [e^\nu, e^\kappa, \cdot] \; x^\lambda \; h(|x|) \; ,$$

many solutions, all like:



$$\mathcal{F} \neq \star\mathcal{F} \text{ AND } \mathcal{F} \neq 0$$

Trick
to satisfy fake curvature

$\mathcal{F} \neq 0$ Lie 2-algebra
becomes $\mathcal{F} = 0, \ \mathcal{H} = 0$ Lie 3-algebra

$\mathfrak{h} \xrightarrow{\mathsf{t}} \mathfrak{g}$
becomes $\mathfrak{h} \xrightarrow{\mathsf{t}} \mathfrak{g} \ltimes \mathfrak{h} \xrightarrow{\mathsf{t}} \mathfrak{g}$

$\mathfrak{su}(2)$-monopole

$$\Phi \sim g^{-1} \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix} g \ .$$

$$\pi_2(\mathsf{SU}(2)/\mathsf{U}(1)) \cong \mathbb{Z} \ .$$

Similarly for A_4 self-dual string

$$\pi_3\left(\mathsf{SU}(2) \times \mathsf{SU}(2)/\mathsf{SU}(2)\right) \cong \mathbb{Z} \ ,$$

Basic (BPST) Instantons

$$F \in \mathfrak{su}(2),$$
$$F = \star F \ ,$$
$$F \to 0 \text{as} |x| \to 0 \ ,$$

$$F = \rho^2 \frac{\mathrm{d}x \wedge \mathrm{d}\bar{x}}{(\rho^2 + |x|^2)^2}$$
$$x = x^\mu \sigma^\mu$$

$\mathrm{d}x \wedge \mathrm{d}\bar{x}$ self-dual on $\mathbb{R}^4$

$$\mathrm{d}x \wedge \mathrm{d}\bar{x} \wedge \mathrm{d}x \text{ self-dual on } \mathbb{R}^{1,5}$$

with $x = x^M \sigma^M =$

$$\begin{pmatrix} 0 & x_0 + x_5 & -x_3 - \mathrm{i}x_4 & -x_1 + \mathrm{i}x_2 \\ -x_0 - x_5 & 0 & -x_1 - \mathrm{i}x_2 & x_3 - \mathrm{i}x_4 \\ x_3 + \mathrm{i}x_4 & x_1 + \mathrm{i}x_2 & 0 & -x_0 + x_5 \\ x_1 - \mathrm{i}x_2 & -x_3 + \mathrm{i}x_4 & x_0 - x_5 & 0 \end{pmatrix} .$$

$$H \propto \mathrm{d}x \wedge \mathrm{d}\bar{x} \wedge \mathrm{d}x$$

many solutions, e.g.:

$$H := \frac{\rho^2}{(\rho^2 + |x|^2)^{\frac{5}{2}}} \begin{pmatrix} 0 & \mathrm{d}\hat{x} \wedge \mathrm{d}x \wedge \mathrm{d}\hat{x} \\ 0 & 0 \end{pmatrix} = \star H$$

$$F = \frac{1}{(\rho^2 + |x|^2)^2} \begin{pmatrix} \rho^2\,\mathrm{d}\hat{x} \wedge \mathrm{d}x + \frac{1}{2}\mathrm{d}\hat{x}\,x \wedge \mathrm{d}\hat{x}\,x & 0 \\ 0 & \rho^2\,\mathrm{d}x \wedge \mathrm{d}\hat{x} + \frac{1}{2}\mathrm{d}x\,\hat{x} \wedge \mathrm{d}x\,\hat{x} \end{pmatrix}$$

In Conclusion

- (1,0) models similar but substantially different from HGT
- (1,0) models include Courant-Dorfman algebras, String Lie 2-algebras and M2-brane 3-algebras
- With fake curvature condition, gauge transformations match Lie 3-algebras but SUSY doesn't fit
- BPS (non-abelian) sector also different from HGT
- Analogues of 't Hooft-Polyakov monopoles and BPST instantons have many solutions

Thank you!